Discrete mathematics

1 Introduction 04/09

- Definition of Graph, Vertex, and Edges
- Complete graph and relation with planar
- Deletion and contraction
  - contraction is equal to deletion in the dual graphs
- What is connected and definition of path and cycle
- definition of subgraph and minor
  - minor is one of the most inmportant concepts in the graph theory
- Planar graph
- Euler formula

2 Basic knowledge of Network 04/16

Today, I learnt how we express network in mathematics, wondering how beautiful theory flow and cut will make.

- definition of network and flow and cut
- prove the following theorem
  1. $$\partial f(s) = -\partial f(t)$$
  2. $$v(f) \leq c(U)$$
  3. Max-flow min-cut theorem
  $$|f_{\max}| = c\left((S,T)_{\min}\right)$$

3 Network and application 04/23

Today I learnt the algorithms to find maximum flow and some applications of network.

- Algorithms that return the maximum flow of the Network
- Definition of the bipartite graph

- Prove the equation between size of maximum matching and size of minimum cover
- Meiger theorem

4 Linear Programming 05/07

- Canonical forms
  - minimize
$$\mathbf{c}^{\mathrm{T}}\mathbf{x}$$
  - under
$$A\mathbf{x} \leq \mathbf{b}$$
$$\mathbf{x} \geq \mathbf{0}$$
-
- simplex method
  - begins at a starting vertex and moves along the edges of the polytope until it reaches the vertex of the optimal solution.
- Pivot operations
- Tableau
- Three cases of simplex method that starts with B
  1. C_N is positive → basis solution is the answer.
  2. c_j and $a_j$is negative → -infinity
  3. $c_j$is negative and a_j is positive → pivoting

  http://college.cengage.com/mathematics/larson/elementary$_linear/4e/shared/downloads/c09s3.pdf$

5 Duality theorem 05/14

primal problem is
$$Minimize\ \mathbf{c}^\top\mathbf{x}\ subject\ to\ A\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0$$
dual problem is
$$Maximize\ \mathbf{b}^\top\mathbf{y}\ subject\ to\ A^\top\mathbf{y} \leq \mathbf{c}$$

- weak duality
  -
$$\mathbf{c}^\top\mathbf{x} \geq \mathbf{b}^\top\mathbf{y}$$
- Duality
  - if primal problem(P) and dual problem(D) are feasible
$$\mathbf{c}^\top\mathbf{x} = \mathbf{b}^\top\mathbf{y}$$
  - P or D is unfeasible then inf

    - P and D are unfeasible
- Bland's rule
    - It stops in finite time, with no loop
- Complementary slackness theorem
    - when x and y are optimal solutions, one of the above inequalities should be equality

6 Algotithm 05/29

- P, NP
    - Computational complexity theory
- Tree width
    - Tree width is a parameter that represents how similar the object is to the tree
    - Tree decomposition
        * Create a new tree (I,F) from a graph (V,E)
        * each node in I is corresponds to the subset of the vertex V
        * connected vertexes in (V,E) are in the same vertex in (I,F).
        * all vertices v in V, vertex includes v are connected in (I,F)
    - Nice tree decomposition makes a graph easy to operate, which enables us to do dynamic programming

7 Shortest path problem 06/11

Today I learnt the use of the dual problem taking an example of the shortest path problems. Using adjacent matrix and law of conservation of flow, we can write the problem as the dual problems. Complementary slackness theorem tells us the condition to be minimum. Seeing it from the view of dijikstra algorithms and other algorithms, we can see the interesting relation with duality.

- summary of the duality
- Shortest path problem
    - Shortest path problem can be seen from the view of the duality

8 Complementary condition 06/18

Today, we picked out the minimum cost flow problem as an example of the dual problems. Minimum cost flow problem is more difficult than shortest path problem, which we learnt in the last class, in terms of the existance of slack variables. Both problems can be written in using matrix and complementary consitions.

- Minimum cost flow

9 The relationships between linear programming and other problems 06/25

Today, we learnt the relation between linear programming and other problems included in the linear programming through examples. After that we learnt the four characteristics of graphs. In the next class, we will learn the relation between them.

- Bipartite and not bipartite graphs
  - In bipartite graph, max matching is equals to the minimum vertex cover.
  - Sloving this problems in linear programming causes an interface with non-integral world
- Four important characteristics of a graph
  - Chromatic Number $\chi(G)$, the smallest number of colors needed to color the vertices
  - The size of maximum clique, $\omega(G)$
  - Maximum independent set, $\alpha(G)$
  - Minimum clique cover, $\theta(G)$

10 Perfect graph 07/02

Today, we learnt a graph theory focused on the vertices

- Complement graph
  - $max w(G) = min \alpha(\overline{G})$
  - $max \chi(G) = min K(\overline{G})$
- relation with weak duality
  - max $w(G) \leq$ min $\chi(G)$
  - max $\alpha(G) \leq$ min $K(G)$
- α perfect and χ perfect
  - graph is said to be α perfect if and only if $\alpha(G_S) = K(G_S)$
  - graph is said to be χ perfect if and only if $(G_S) = \chi(G_S)$
- interval graph

11 Matroid 07/09

- Four matroid axioms
  - independent axioms
  - rank axioms
  - base axioms
  - circuit axioms
- Linear matroid
- Graphic matroid

- Dual matroid