Mathematics 1D

Impression

- I took this class because I want to know the differential equation and the vector analysis as the basic knowledge of mathematics
- This class has a lot to master
- Proof makes me understand better
- I did not totally unserstand rotation and divergence a year ago, but finally I master it. But I do not think it is useful in the computer science.

Memo

1. Differential equation
   - Logistic equation
     - separate variables
     - sometimes use partial fraction decomposition
   - Liouville's equation
     - $y'' + P(x) * y' + Q(y)(y')^2 = 0$
     - divide by $y\prime$ and integrate it
   - $dy/dx = f(dy/dx)$
     - let $u = y/x$
     - scale conversion
     - $dy/dx = xy/(x^2 + y) \rightarrow (x = \lambda x, y = \lambda^2 y)$
     - $dy/dx = x^2 y/(x^3 + y) \rightarrow (x = \lambda x, y = \lambda^3 y)$
     - variable conversion $dy/dx = (x - 2y + 3)/(2x + y - 4)$
   - variation of parameters
     - $y' + P(x)y = Q(x)$
     - let $Q(x) = 0$ and solve the equation, then let constant be the function of x, C(x)
     - y = $\mathrm{e}^{\int -P(x)dx}(\int Q(x)\mathrm{e}^{\int P(x)dx}dx + C)$
   - Bernoulli differential equation
     - $y' + P(x)y = Q(x)y^m$
     - let $z = y^{1-m}$
   - Riccati equation
     - find particular solution $y = f(x)$, then let z be $y - f(x)$

- D'Alembert's equation
  - $y = xf(y') + g(y')$
  - let $P = y'$, then deriviate the equation with respect to x.
  - Clairaut equation
- Total differential equation
  - $P(x, y)dx + Q(x, y)dy$
  - Exact differential equation $(\partial F/\partial x = P(x, y), \partial F/\partial y = Q(x, y)) \rightarrow$ solution is $F(x, y) = C$
  - Integrating factor is a factor that make total differential equation be exact differential equation.
- Second-order homogeneous differential equation
  - $x'' + 2\gamma x' + w^2 x = 0$
  - $y = C_1 \mathrm{e}^{px} + C_2 \mathrm{e}^{qx}$ (when p and q are real solutions)
  - $y = \mathrm{e}^{hx}(C_1 coskx + C_2 sinkx)$ (when h+ki and h-ki are imaginary solutions)
  - $y = \mathrm{e}^{px}(C_1 + C_2 x)$ (when heavy solution)
- n-order homogeneous differential equation
  - $y = \Sigma C_k \mathrm{e}^{\lambda_k x}(\lambda_k$ is a solution)
  - $y = \Sigma\Sigma C_m^k x^m e^{\lambda_k x}$
- n-order inhomogeneous differential equation (with respect to find a particular sollution)
  - $\left(\sum_{k=1}^{n} c_k \frac{\mathrm{d}^k}{\mathrm{d}x^k}\right) u(x) + c_0 u(x) = f(x)$
  - substitute $Ce^{\alpha x}$ or $Cx^m e^{\alpha x}$ for the quation when $f(x) = e^{\alpha x}$
  - substitute n th order polyomial when f(x) is n th order polyomial
  - solution is (particular solution) + (general solution)
- Higher-order differential equation
  - $y^{(n)} + p_{n-1}(t)y^{(n-1)} + \cdots + p_1(t)y' + p_0(t)y = 0$
  - how to write above equation using matrix and vector.
  - let vector $\boldsymbol{u} = \begin{pmatrix} y^{(n-1)} \\ y^{(n-2)} \\ \vdots \\ y \end{pmatrix}$
  - detect A s.t. $(\boldsymbol{u'} = A\boldsymbol{u})$
  - A will be like a transpose of Jordan block
- Two dimensional systems of differential equation
  - $\frac{d\vec{\mathbf{x}}}{dt} = A\vec{\mathbf{x}}$
  - get the eigenvalues (λ_1, λ_2) and get the eigenvectors($\boldsymbol{v}_1$, $\boldsymbol{v}_2$).
  - general solution is $\boldsymbol{x}(t) = C_1 e^{\lambda_1 t}\boldsymbol{v}_1 + C_2 e^{\lambda_2 t}\boldsymbol{v}_2$
  - Use generalized eigenvector, if the characteristic polynomial has only one eigenvalue
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2. Vector calculus
   - inner product and outer product
   - gradient

    - $\nabla \cdot f$
    - the gradient of scalar function at that point will show the direction in which the value rises most quickly. Plus it can measure how scalar field changes in other direction by using dot product
  - divergence
    - $\nabla \cdot \boldsymbol{F}$
    - scalar valued function
    - motivation is knowing "flowing towards and away"
  - rotation
    - $\nabla \times \boldsymbol{F}$
    - vector potential is like the flow of water
    - rotation is a like the power that is given by the flow.
  - line element
  - Green's theorem
  - Stokes' theorem
  - Gauss's theorem
3. Variational method
  - Euler-equation
  - Beltrami
  - Euler-Lagrange

日本語での感想

- 理系なのに微分方程式やベクトル解析ができないのは渋かったので履修
- 証明もきちんと行っていただいたのですごく理解できた
- 他学科の人はこの授業内容と同じ内容の授業があるらしく、楽そうだったが、結構分量が多くてハードだった
- 途中からはベクトル解析使わなくないか、という疑問が絶えなかった